Adding police to a mathematical model of burglary

Funded by: NSERC, ORS Award

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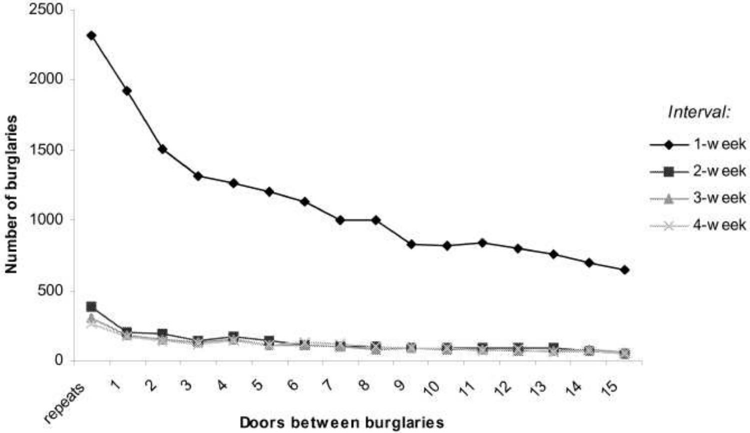
Workshop on Mathematical Models of Urban Criminality
Pisa 2008

Why burglary?

- Highly reported $\therefore$ good data
- Patrols have a greater effect on deterring property crime than violent crime
- Data shows burglaries are highly spatially and temporally clustered
    - Repeat victimisation
    - Near-repeat victimisation

Evidence from data for the Boost Hypothesis

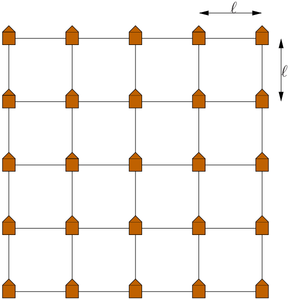


Overview of the first model

(MB Short, MR D'Orsogna, VB Pasour, GE Tita, PJ Brantingham, AL Bertozzi, LB Chayes)

Each house is described by its lattice site $s = (i, j)$ and a quantity $A_s(t)$ (attractiveness).

$$A_s(t) = A_s^0 + B_s(t) > 0$$



Probability a burglar commits a burglary:

$$p_s(t) = 1 - e^{-A_s(t)\delta t}$$

During each time interval δt, burglars must perform exactly one of the following two tasks:

1. Burgle the home at which they are currently located, or
2. move to one of the adjacent homes (biased towards high $A_s(t)$).

n_s = number of burglars at site s

When a house is burgled:

- The corresponding burglar is removed from the lattice.
- B_s is increased by a quantity θ then decays over time.

$$B_s(t+\delta t) = B_s(t)(1-\omega_1\delta t) + \theta n_s(t)p_s(t) \quad (1)$$

Near-repeat victimisation is modelled by allowing $B_s(t)$ to spread to its neighbours.

$$B_s(t+\delta t) = \left((1-\eta)B_s(t) + \frac{\eta}{4}\sum_{s'\sim s} B_{s'}(t)\right)(1-\omega_1\delta t) + \theta n_s(t)p_s(t) \quad (2)$$

which can be written in terms of the discrete spatial Laplacian as

$$B_s(t+\delta t) = \left(B_s(t) + \frac{\eta\ell^2}{4}\Delta B_s(t)\right)(1-\omega_1\delta t) + \theta n_s(t)p_s(t), \quad (3)$$

where

$$\Delta B_s(t) = \left(\sum_{s'\sim s} B_{s'}(t) - 4B_s(t)\right)/\ell^2. \quad (4)$$

- Burglars come from sites they did not burgle at in the previous time step
- Burglars are generated at each site at a rate Γ

$$n_s(t+\delta t) = A_s(t) \sum_{s' \sim s} \frac{n_{s'}(t)[1 - p_{s'}(t)]}{T_{s'}(t)} + \Gamma \delta t, \tag{5}$$

where

$$T_{s'}(t) := \sum_{s'' \sim s'} A_{s''}(t) = 4A_{s'}(t) + \ell^2 \Delta A'_s(t). \tag{6}$$

- Convert n_s to a density ρ_s by dividing it by ℓ^2

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Continuum equations

Let $\delta t \to 0$ and $\ell \to 0$ with the constraint that $\ell^2/\delta t$ tends to the constant D
and $\epsilon = \theta\delta t$ and $\gamma = \Gamma/\ell^2$.

Then the continuum equations corresponding to dynamic burglary attractiveness and burglar density are:

$$\begin{aligned}
\frac{\partial B}{\partial t} &= \frac{\eta D}{4}\nabla^2 B - \omega_1 B + \epsilon D\rho(A^0 + B), && (7)\\
\frac{\partial \rho}{\partial t} &+ \frac{D}{4}\boldsymbol{\nabla}\cdot\left(\frac{2\rho}{A^0+B}\boldsymbol{\nabla} B\right) = \frac{D}{4}\nabla^2\rho - \rho(A^0+B) + \gamma, && (8)
\end{aligned}$$

where A^0 is assumed to be spatially uniform.

Nondimensionalisation

The natural timescale is $1/\omega_1$, the time it takes for the dynamic attractiveness of a newly burgled home to return to its baseline value. Using

$$t = \frac{1}{\omega_1} t', \quad x = \frac{1}{2}\sqrt{\frac{D}{\omega_1}} x', \quad y = \frac{1}{2}\sqrt{\frac{D}{\omega_1}} y',$$

$$A = \omega_1 (A^0 + B)', \quad \rho = \frac{\omega_1}{\epsilon D} \rho',$$

and, dropping primes, we have the nondimensional system

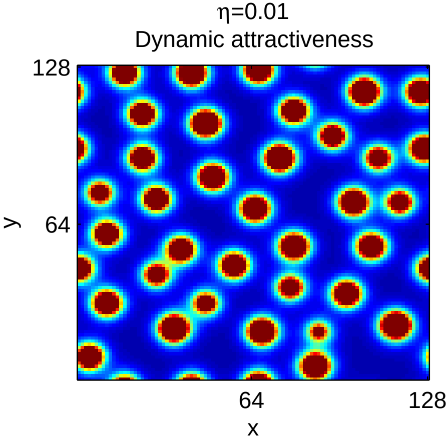
$$\begin{aligned} \frac{\partial B}{\partial t} &= \eta \nabla^2 B - B + \rho(A^0 + B), \\ \frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \left(\frac{2\rho}{A^0 + B} \boldsymbol{\nabla} B \right) &= \nabla^2 \rho - \rho(A^0 + B) + \overline{B}, \end{aligned}$$

where $\overline{B} = \dfrac{\epsilon D \gamma}{\omega_1^2}$ (the homogeneous equilibrium).

Numerical solution

Initial conditions: $\rho(\mathbf{x}, 0) = \overline{\rho}$, and similary for B except a few cells start with values slightly higher than $\overline{B}$.

Boundary conditions: periodic



Police deterrence

- Visible police presence causes a decrease in local crime
- Crime displacement or diffusion of benefits?

Let $d_s(t)$ be the reduction in the statistical rate of burglary when a burglar is present.
Then

$$p_s(t) = 1 - e^{-\left(A_s(t) - d_s(t)\right)\delta t}. \tag{9}$$

Then the equation for the dynamic burglary attractivness becomes

$$\frac{\partial B}{\partial t} = \frac{\eta D}{4}\nabla^2 B - \omega_1 B + \epsilon D \rho (A^0 + B - d). \tag{10}$$

High $d_s(t)$ relative to neighbouring sites should reduce the chance of burglars moving there in the next time step. Then the discrete equation for the number of burglars at site s becomes

$$n_s(t+\delta t) = \big(A_s^0 + B_s(t) - d_s(t)\big) \sum_{s' \sim s} \frac{n_{s'}(t)[1 - p_{s'}(t)]}{T_{s'}(t)} + \Gamma \delta t, \quad (11)$$

where

$$T_{s'}(t) := \sum_{s'' \sim s'} \big(A_{s''}^0 + B_{s''}(t) - d_{s''}(t)\big). \quad (12)$$

Then the corresponding continuum equation for burglar density is

$$\frac{\partial \rho}{\partial t} + \frac{D}{4} \boldsymbol{\nabla} \cdot \left(\frac{2\rho}{A^0 + B - d} \boldsymbol{\nabla}(B - d) \right) = \frac{D}{4} \nabla^2 \rho - \rho(A^0 + B - d) + \gamma. \quad (13)$$

Discrete equation for dynamic deterrence (d)

If we assume that police presence causes a deterrent effect as well as a diffusion of benefits to neighbouring sites, then we can write the following discrete equation for the dynamic deterrence:

$$d_s(t+\delta t) = \left[(1-\zeta)d_s(t) + \frac{\zeta}{4}\sum_{s'\sim s} d_{s'}(t)\right](1-\omega_2\delta t) + \xi U_s(t), \qquad (14)$$

which can be written in terms of the discrete spatial Laplacian as

$$d_s(t+\delta t) = \left[d_s(t) + \frac{\zeta\ell^2}{4}\Delta d_s(t)\right](1-\omega_2\delta t) + \xi U_s(t). \qquad (15)$$

Note that this still incorporates the possibility of some displacement of burglary since burglars who choose not to burgle at a site s, move on to an adjacent site and may burgle there in the next time step.

Continuum equation for dynamic deterrence

Subtract $d_s(t)$ from both sides of the discrete equation, convert $U_s(t)$ to a density $u_s(t)$, divide by δt, and take the limit as $\delta t, \ell \to 0$ such that that $\ell^2/\delta t$ tends to the constant D.

Then the continuum equation for dynamic deterrence is

$$\frac{\partial d}{\partial t} = \frac{\zeta D}{4} \nabla^2 d - \omega_2 d + \xi D u. \tag{16}$$

New p.d.e. system

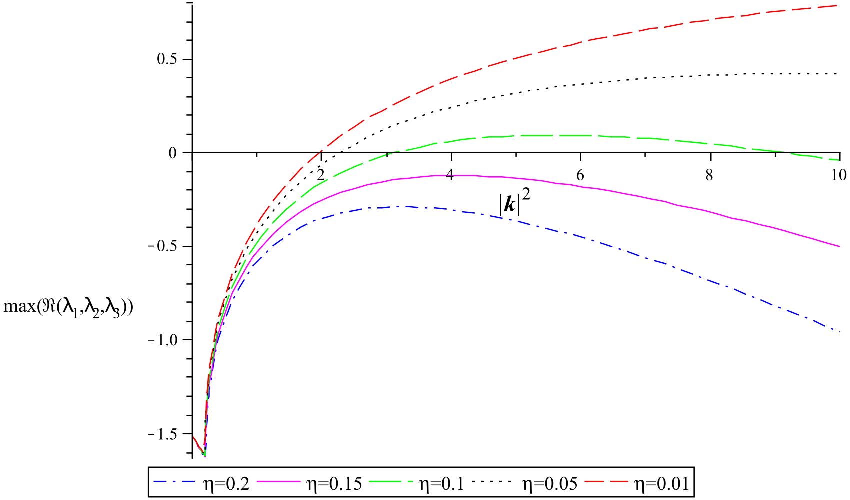
We nondimensionalise as previously to obtain the nondimensional system

$$\frac{\partial B}{\partial t} = \eta \nabla^2 B - B + \rho(A^0 + B - d),$$
$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \left(\frac{2\rho}{A^0 + B - d} \boldsymbol{\nabla}(B - d) \right) = \nabla^2 \rho - \rho(A^0 + B - d) + \overline{B},$$
$$\frac{\partial d}{\partial t} = \zeta \nabla^2 d - \omega d + u,$$

where $\omega = \omega_2/\omega_1$ and assume $A^0 + B - d > 0$.

First, assume constant police patrol strategy u.

Stability to spatial disturbances



What's next?

- Look at different police patrol strategies u
- Look at relaxing some of our assumptions:
  - uniformly distributed housing
  - spatially constant burglar generation rate (Γ) and static burglary attractiveness (A^0)
  - road network and side-of-the-street effects ignored
- Determine optimal patrol strategies

Optimal control formulation

$$\min_u J := \int_0^T \int_\Omega \rho \left(A^0 + B - d\right) \mathrm{d}A\mathrm{d}t, \tag{17}$$

subject to the p.d.e.s

$$\begin{aligned}
\frac{\partial B}{\partial t} &= \eta \nabla^2 B - B + \rho(A^0 + B - d), \\
\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \left(\frac{2\rho}{A^0 + B - d} \boldsymbol{\nabla}(B - d) \right) &= \nabla^2 \rho - \rho(A^0 + B - d) + \overline{B}, \\
\frac{\partial d}{\partial t} &= \zeta \nabla^2 d - \omega d + u,
\end{aligned}$$

and the constraint

$$\int_\Omega u \,\mathrm{d}A = R, \quad \forall t, \tag{18}$$

to represent the officers available for preventative police patrols.